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CHANGE THE WORLD FROM HERE

Networks and Graphs

Basics and Metrics

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Motivation

- **Connectivity**
  - When do we want to **preserve** connectivity?
  - When do we want to **break** connectivity?
- **Robustness**
  - When do we want the network to be robust to **random** failure?
  - When do we want the network to be robust to **targeted** failure?

Slides adapted from http://mae.ucdavis.edu/dsouza/mae298

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BASICS

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Networks and Graphs

- G = (V, E) where
  - V is a set of vertices or nodes
  - E is a set of edges between vertices
- Representation
  - Adjacency matrix
  - Adjacency list
- Used to model networks

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Types of Networks

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Types of Networks

- Computer networks
  - Physical (e.g. Internet)
  - Logical (e.g. WWW)
- Social networks
  - See TED talk…
- Transportation networks
  - Flight paths
  - Interstate system
- Citation networks
- Biological networks
  - Gene regulation networks
  - Signal transduction pathways
- System call graph
- And many more…

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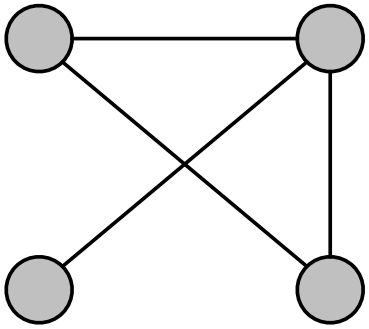
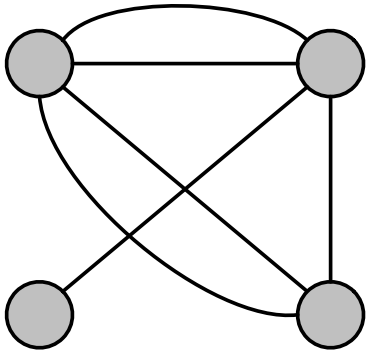
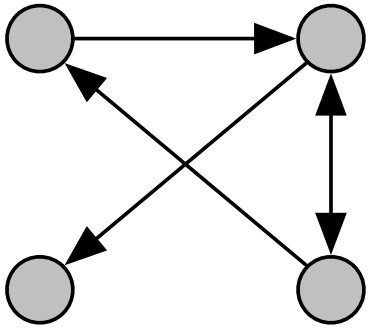
Types of Graphs



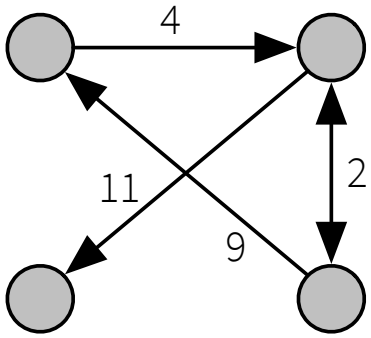
undirected graph



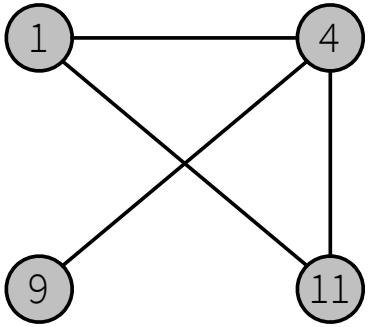
multigraph



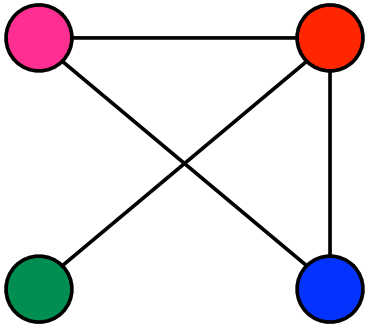
directed graph



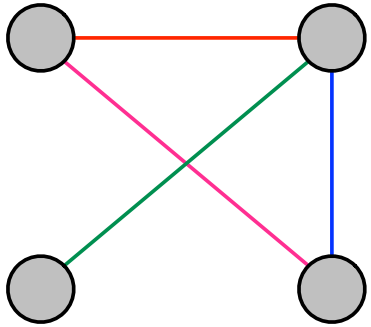
edge-labeled



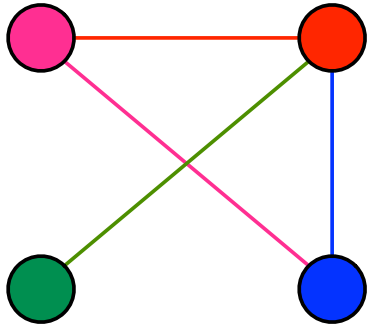
vertex-labeled



vertex-colored



edge-colored



vertex & edge colored

http://mathworld.wolfram.com/Graph.html

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Graph Metrics

- Number of nodes
- Number of edges
- In degree
- Out degree
- Centrality
- Distance
- Vertex cover
- Cut set
- Shortest path
- Page rank
- And more…

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Disciplines

- Graph Theory
  - Works with arbitrary, abstract graphs
  - Deals with famous problems
    e.g. traveling salesman
  - Focus on algorithms and complexity
- Network Theory, Network Science
  - Study real-world representations
    e.g. social networks
  - Focus on network properties/metrics
  - Combines graph theory and social science

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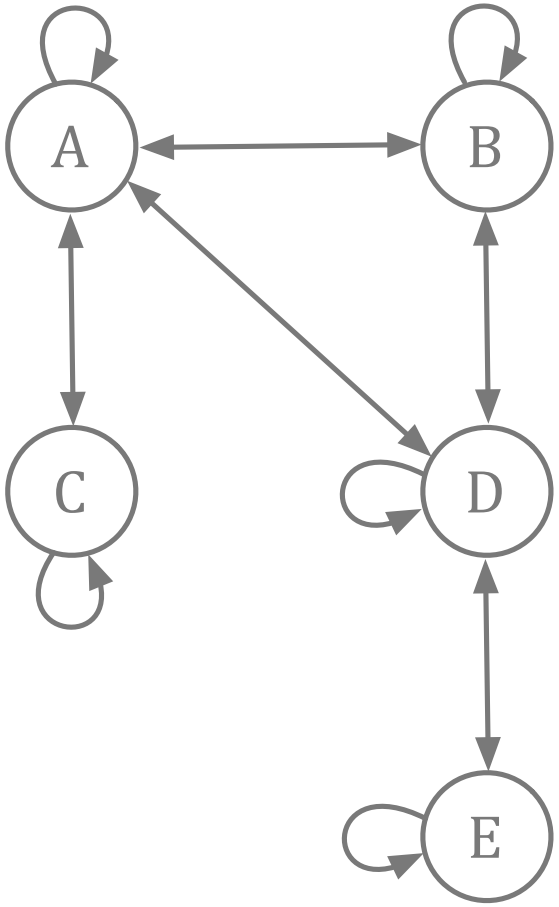
Adjacency Matrix

	A	B	C	D	E
A					
B					
C					
D					
E					



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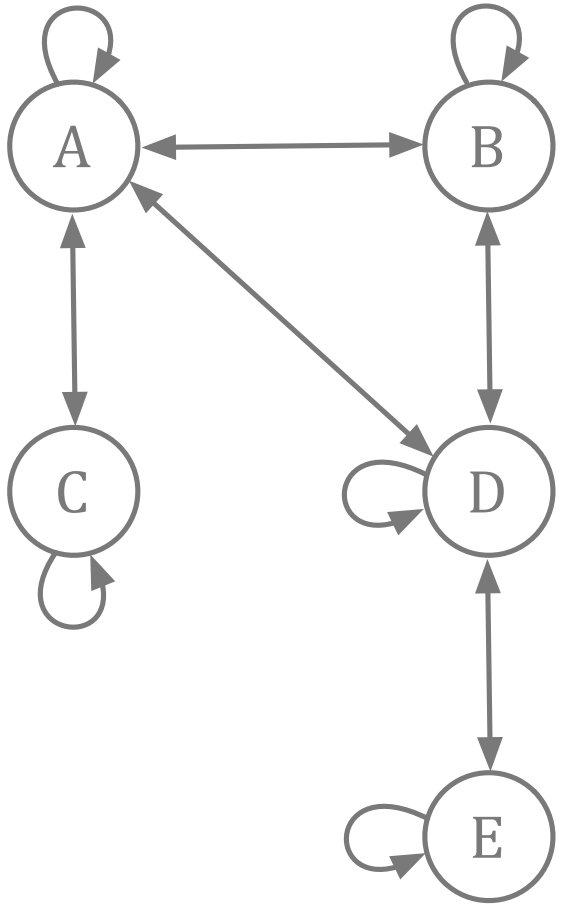
Adjacency Matrix

	A	B	C	D	E
A	1	1	1	1	0
B	1	1	0	1	0
C	1	0	1	0	0
D	1	1	0	1	1
E	0	0	0	1	1



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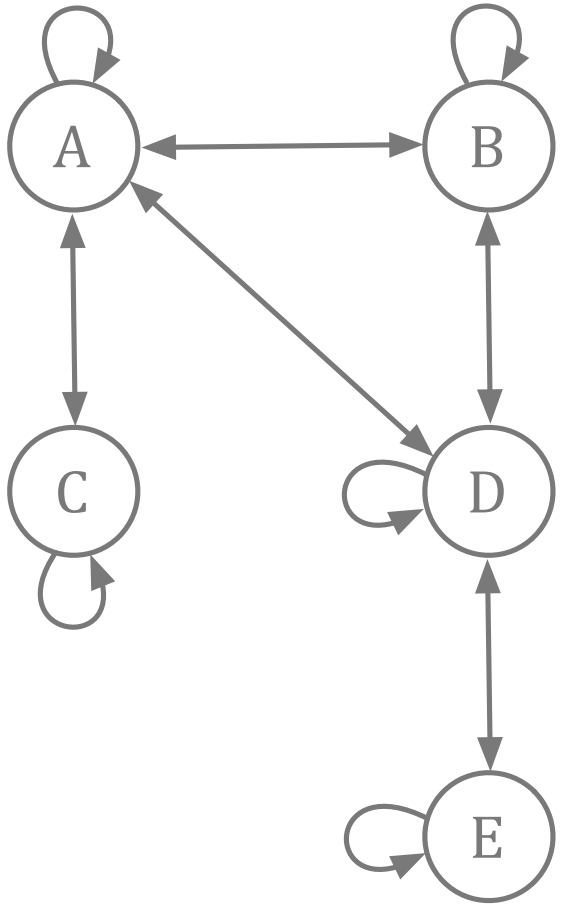

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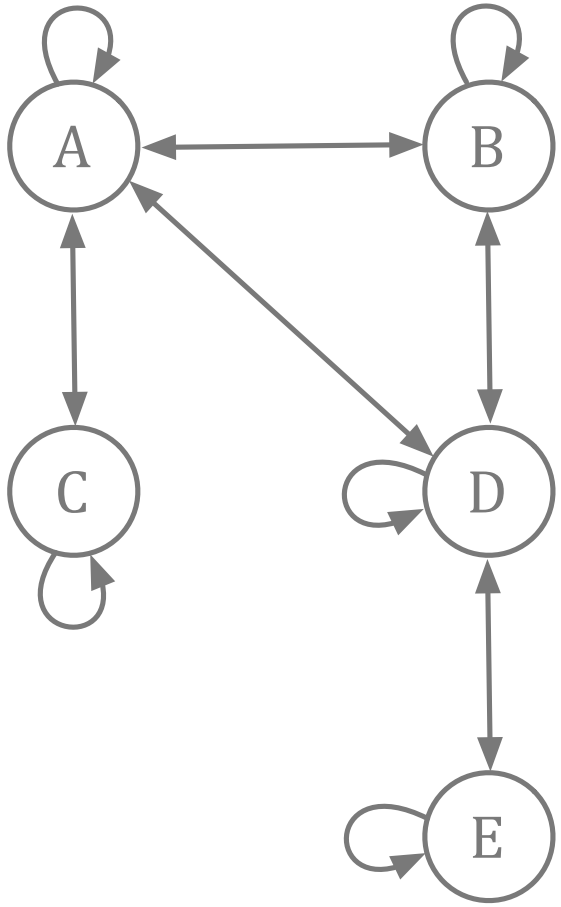
Degree Distribution

	A	B	C	D	E	
A	1	1	1	1	0	4
B	1	1	0	1	0	3
C	1	0	1	0	0	2
D	1	1	0	1	1	4
E	0	0	0	1	1	2
	4	3	2	4	2	



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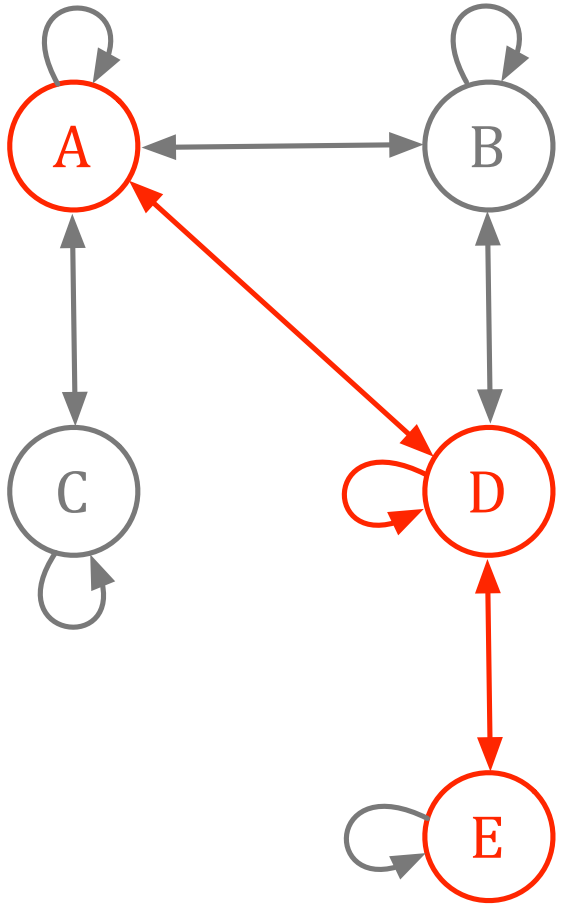
State Transition Matrix

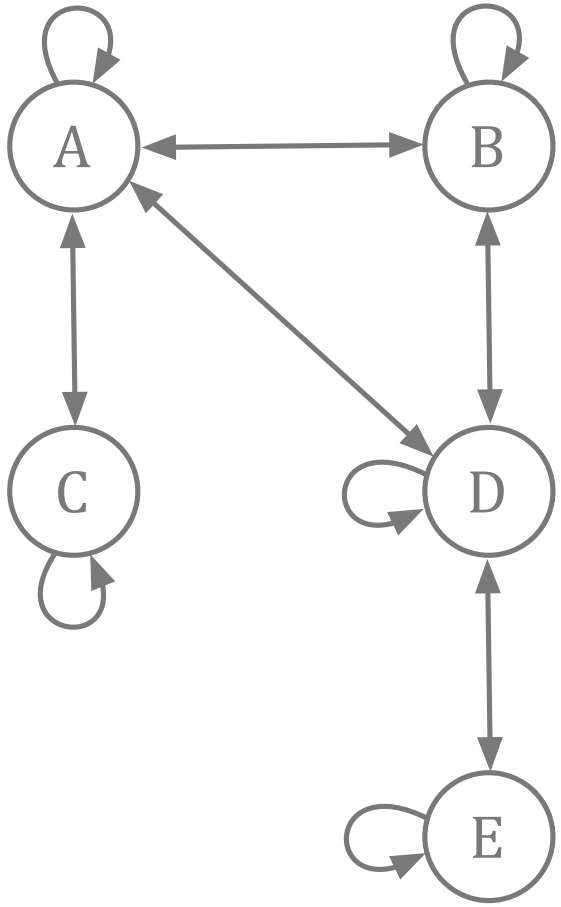
- $P(u, v) = 1 / d_u$
  - Probability that from node *u* you will move to node *v*
- Captures a random walk on the network
  - e.g. *A D D E*



Slides adapted from http://mae.ucdavis.edu/dsouza/mae298

State Transition Matrix

	A	B	C	D	E
A					
B					
C					
D					
E					
	4	3	2	4	2



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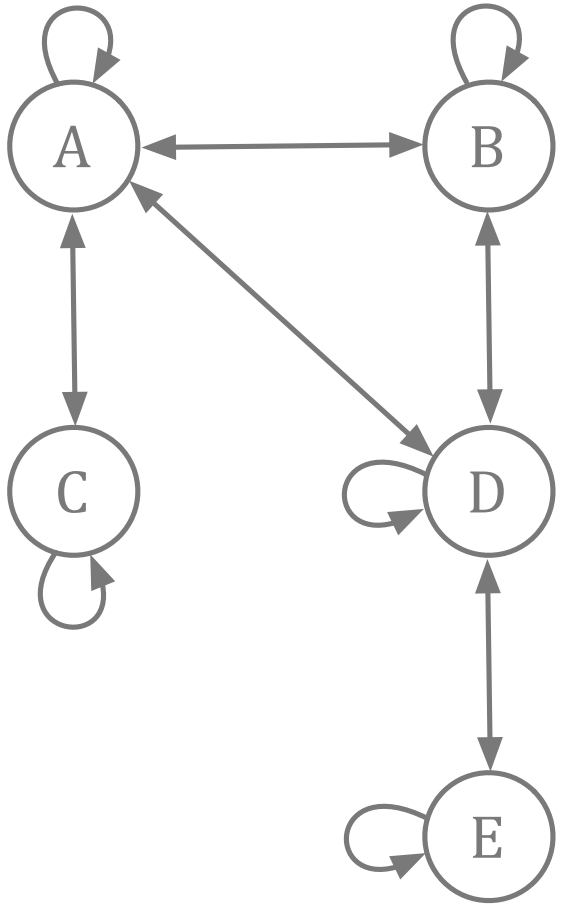
State Transition Matrix

	A	B	C	D	E
A	¼	⅓	½	¼	0
B	¼	⅓	0	¼	0
C	¼	0	½	0	0
D	¼	⅓	0	¼	½
E	¼	0	0	¼	½
	4	3	2	4	2



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Page Rank Algorithm

	A	B	C	D	E	∑
A	¼	⅓	½	¼	0	**1.33**
B	¼	⅓	0	¼	0	0.83
C	¼	0	½	0	0	0.75
D	¼	⅓	0	¼	½	1.33
E	¼	0	0	¼	½	0.75
	4	3	2	4	2	

- Page rank measures relative importance
- Page rank depends on rank of other pages linking to it
- Simple algorithm
  - d_j degree of j
  - $PR(i) = \sum PR(j) / d_j$

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CENTRALITY

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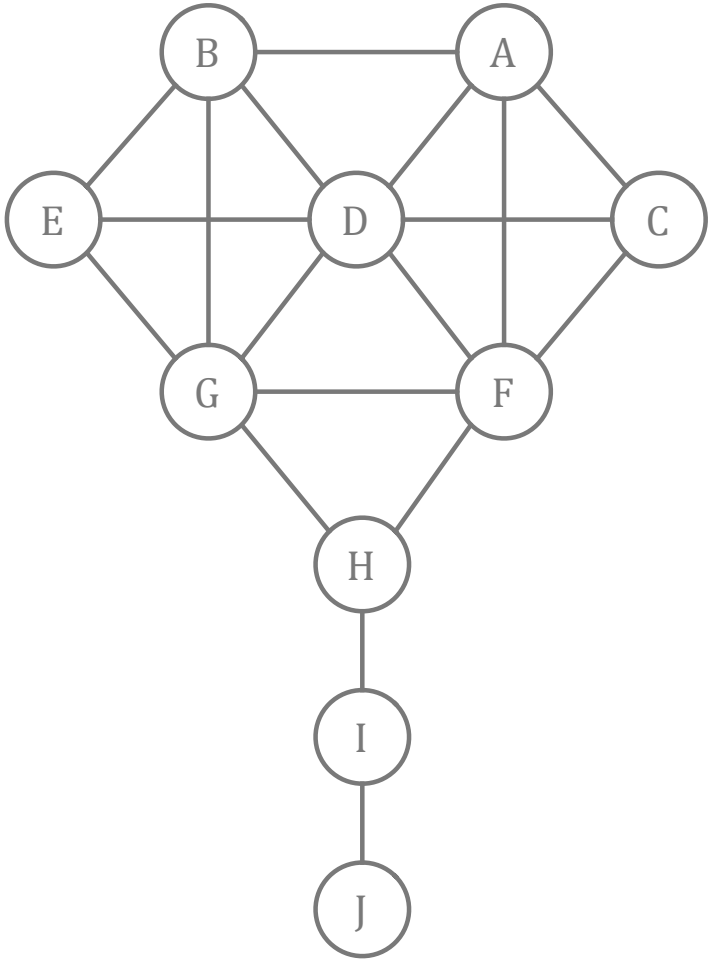
The Kite Network



- Classic example by David Krackhardt
- Models a social network
  - Nodes are people
  - Edges are connections
- Who in the network is important, and why?

http://www.orgnet.com/sna.html

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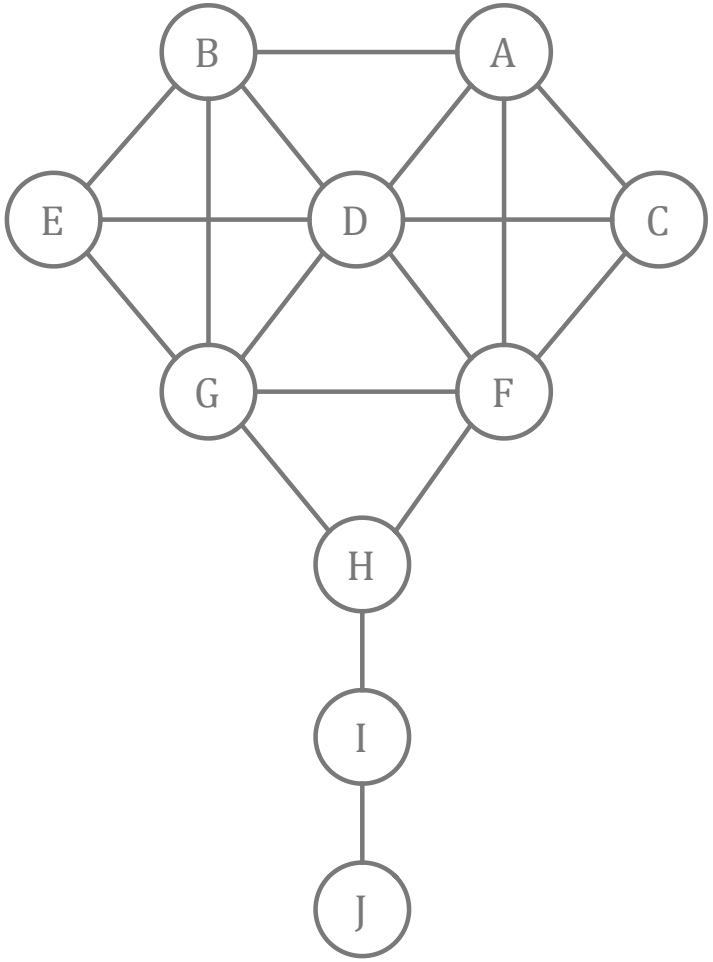
Node Degree



- Andre
- Beverly
- Carol
- Diane
- Ed
- Fernando
- Garth
- Heather
- Ike
- Jane

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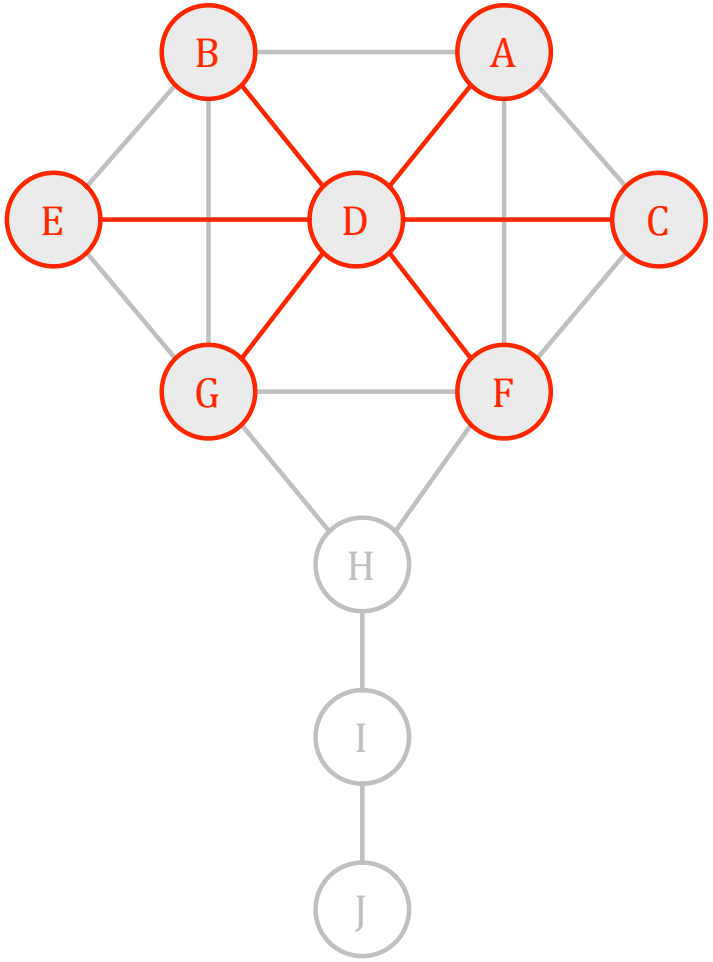
Node Degree



- 6: Diane
- 5: Fernando
- 5: Garth
- 4: Andre
- 4: Beverly
- 3: Carol
- 3: Ed
- 3: Heather
- 2: Ike
- 1: Jane

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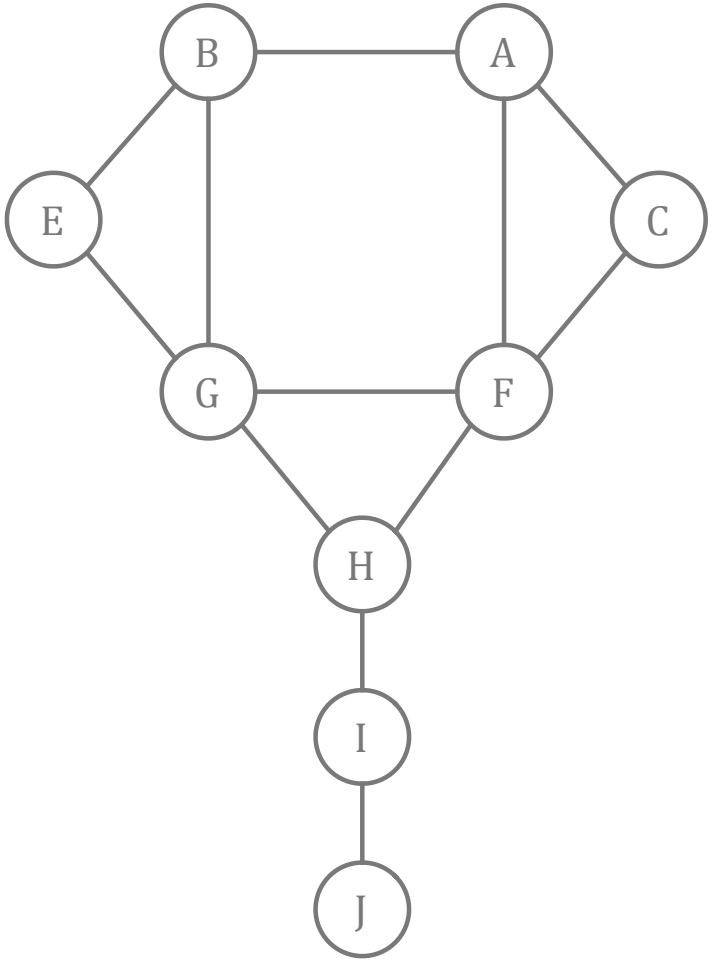

- ~~6: Diane~~
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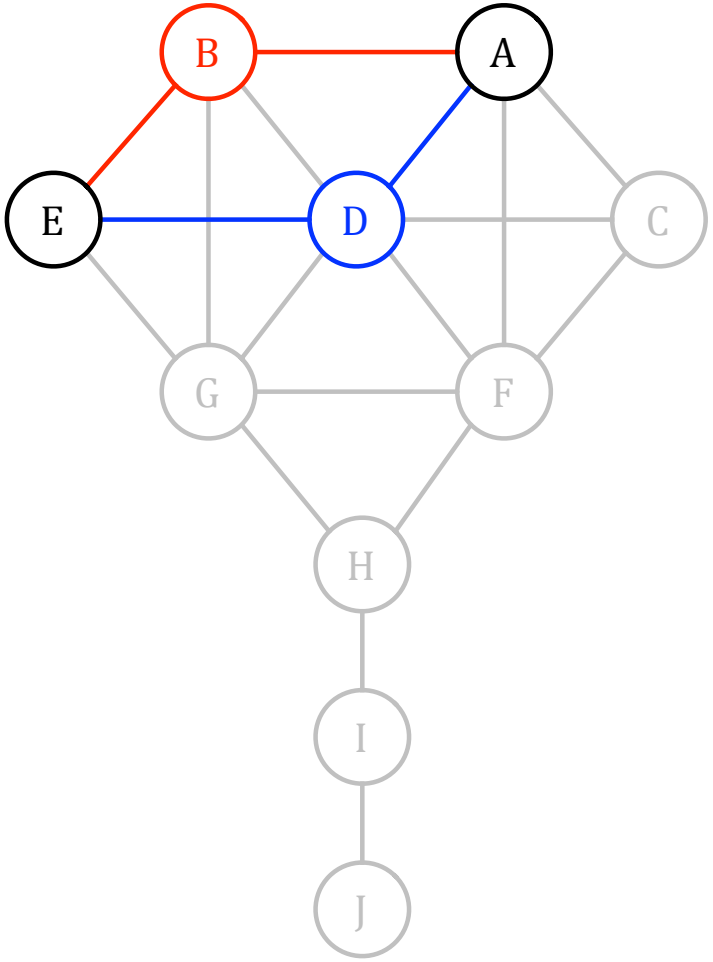
Betweenness Centrality



- Who is "between" the most communities?
- Depends on shortest paths between nodes
- Shortest paths (*E*, *A*)
  - Node *E* to *D* to *A*
  - Node *E* to *B* to *A*

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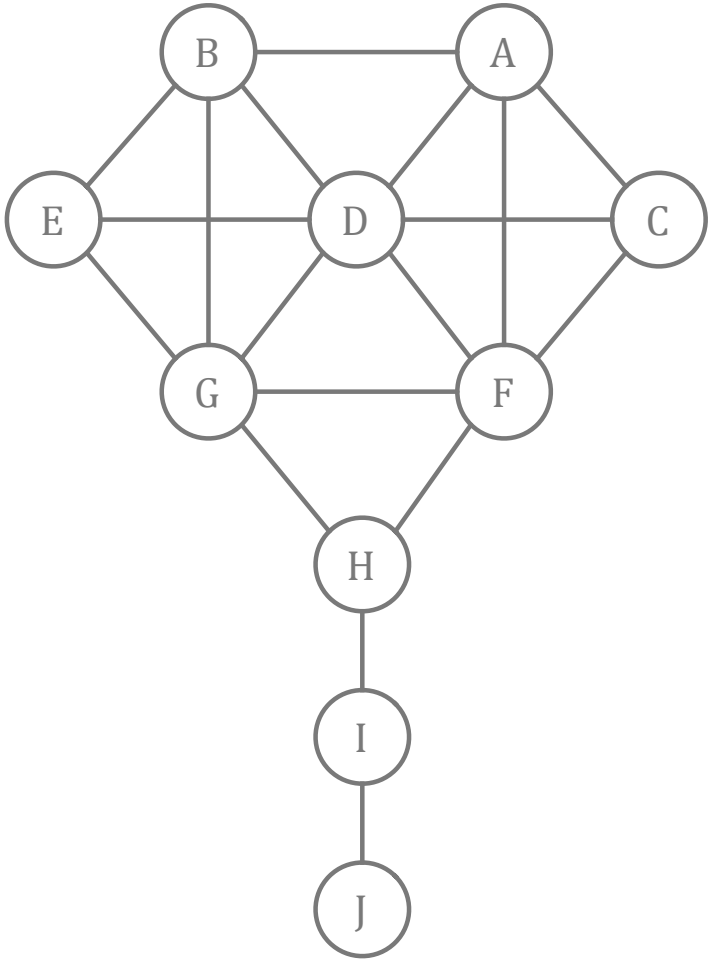
Betweenness Centrality



- For each pair (*s*, *t*)
  - Compute all shortest paths between *s* and *t*
- For each vertex *v*
  - Determine fraction of shortest paths that pass through $v \neq s, t$
  - Sum all fractions to find betweenness

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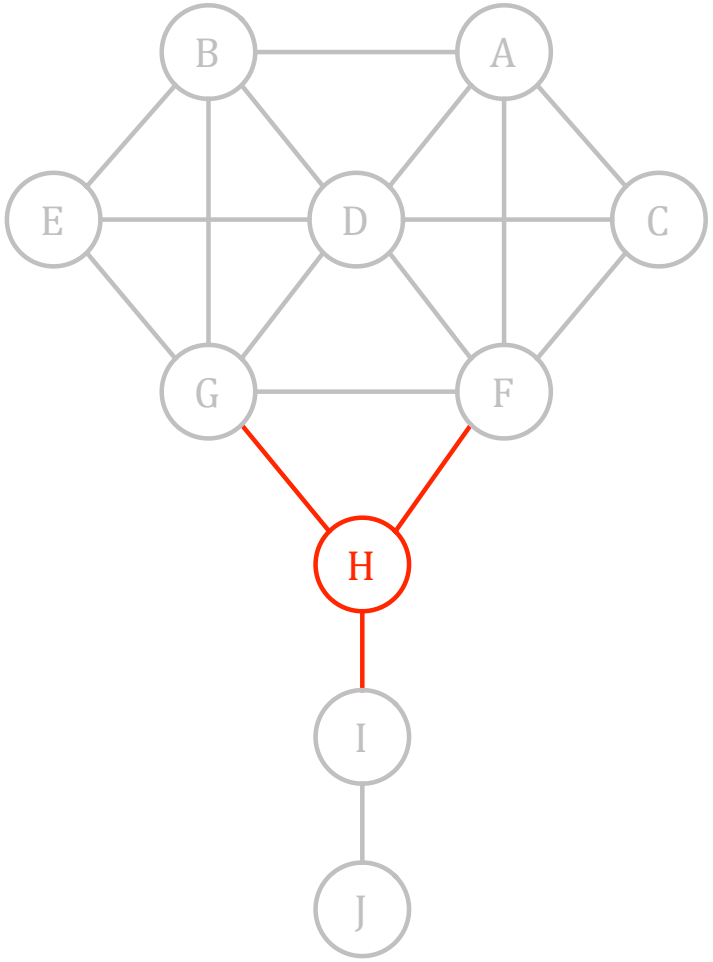

- 0.389: Heather
- 0.231: Fernando
- 0.231: Garth
- 0.222: Ike
- 0.102: Diane
- 0.023: Andre
- 0.023: Beverly
- 0.000: Carol
- 0.000: Ed
- 0.000: Jane

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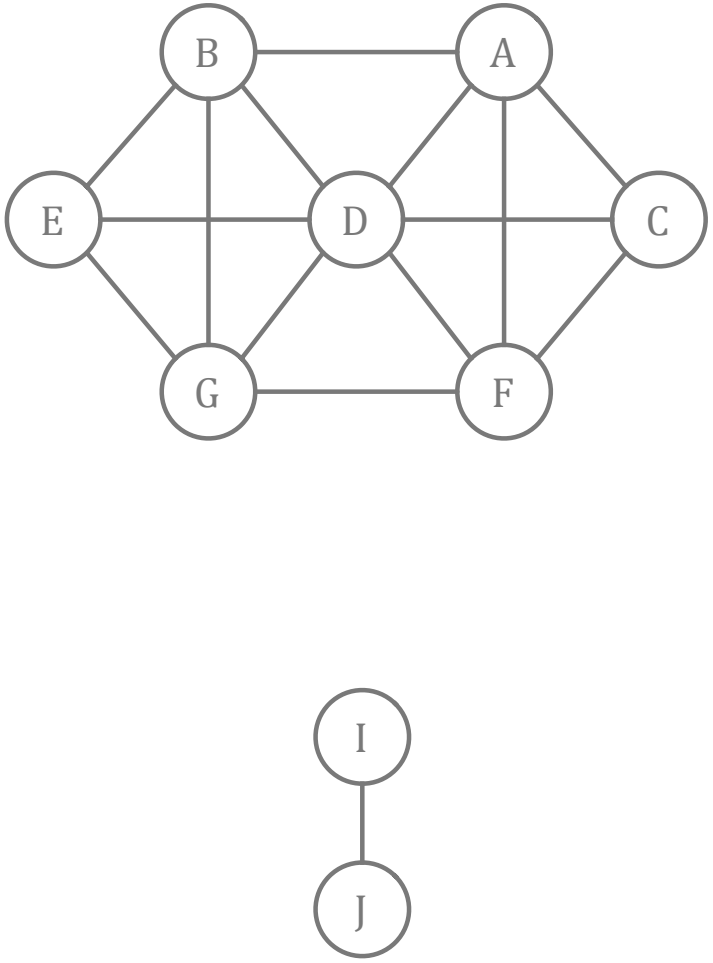
Betweenness Centrality



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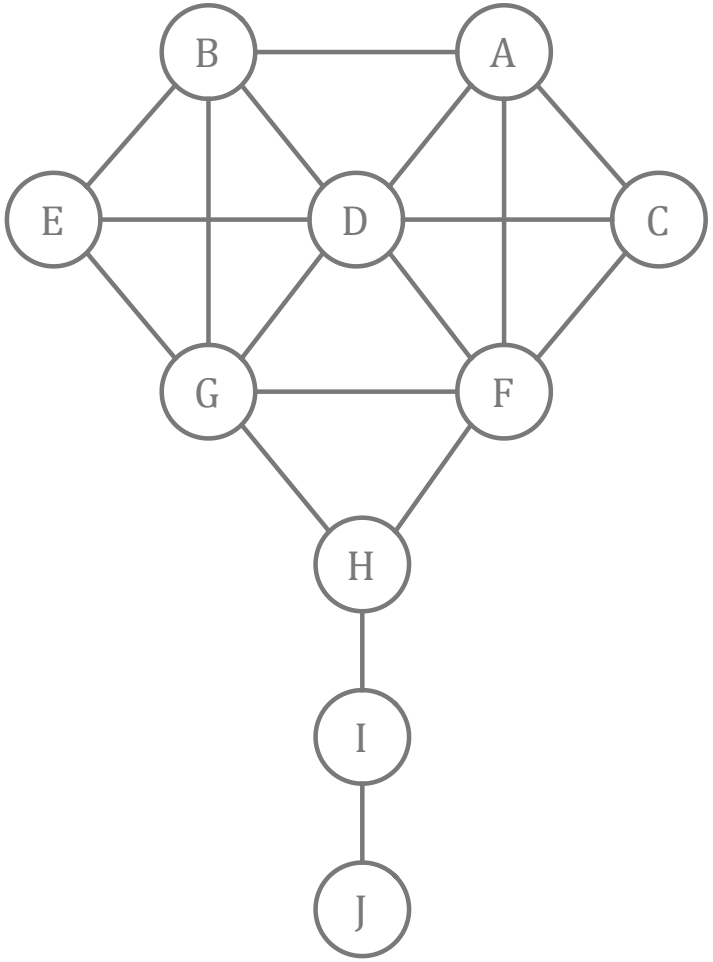
Closeness Centrality



- How "close" other nodes are to node in question
- Distance $d(s, t)$ is the shortest path between nodes s and t
- Computed by average distance between v and all other nodes
  - $(n - 1) \div \sum d(s, t)$

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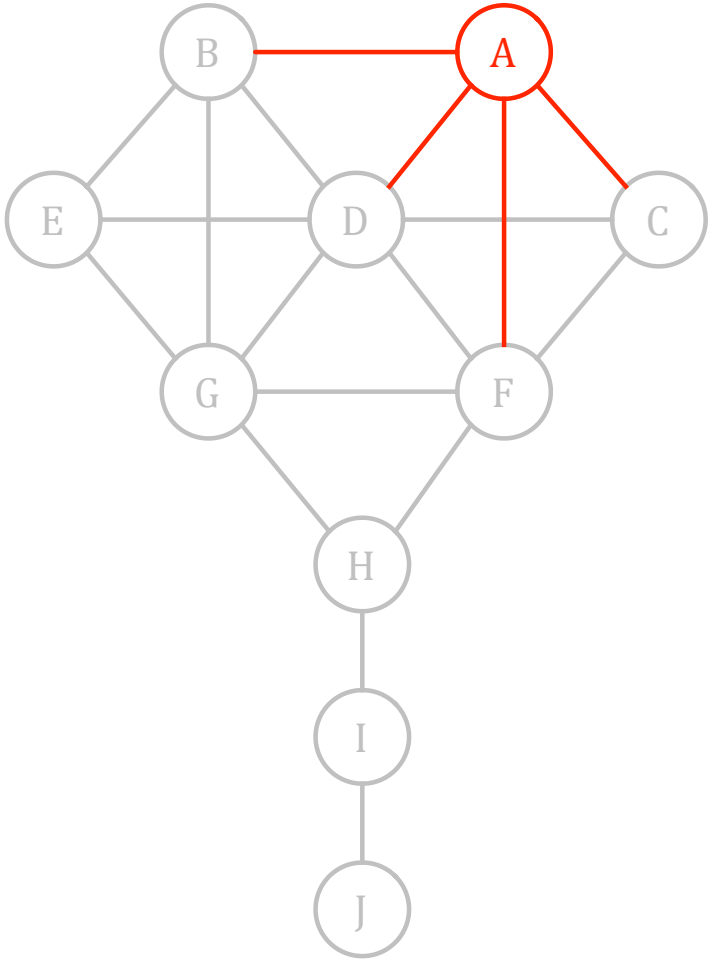
Closeness Centrality



- A to A
- A to B
- A to C
- A to D
- A to E
- A to F
- A to G
- A to H
- A to I
- A to J

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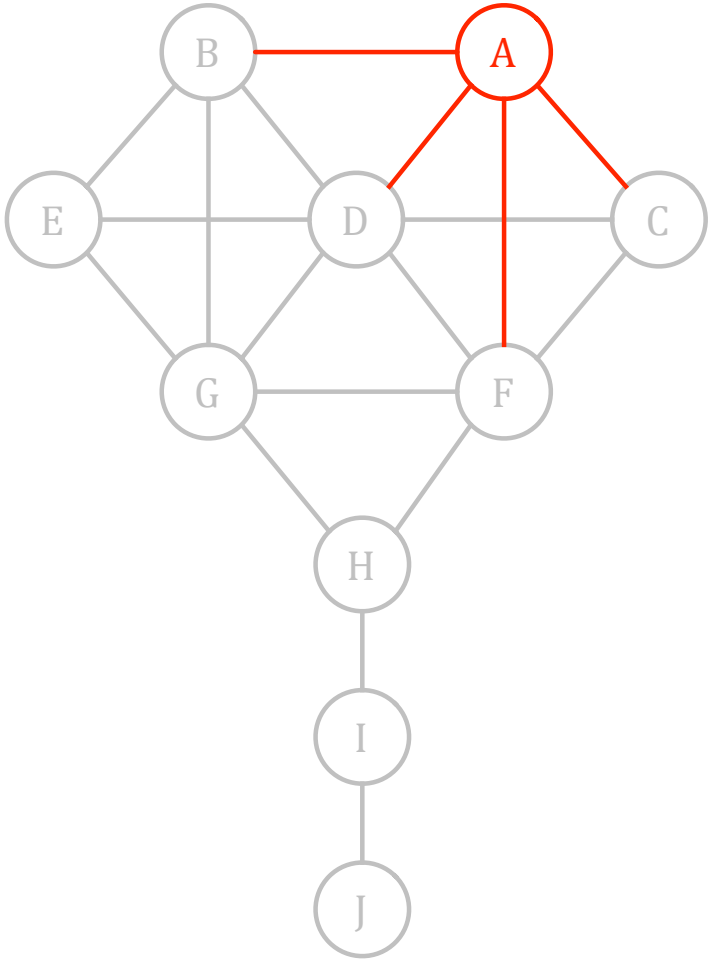
Closeness Centrality



- 1: A to B
- 1: A to C
- 1: A to D
- 1: A to F
- 2: A to E
- 2: A to G
- 2: A to H
- 3: A to I
- 4: A to J
- Average: 9 ÷ 17 = 0.529

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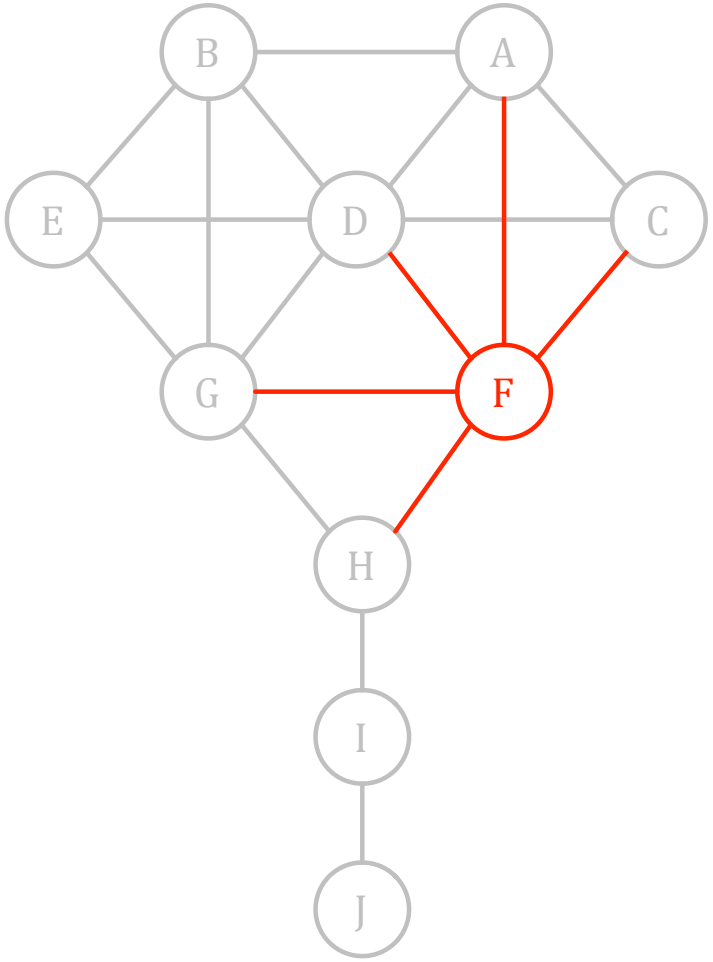
Closeness Centrality



- 0.643: Fernando
- 0.643: Garth
- 0.600: Diane
- 0.600: Heather
- 0.529: Andre
- 0.529: Beverly
- 0.500: Carol
- 0.500: Ed
- 0.429: Ike
- 0.310: Jane

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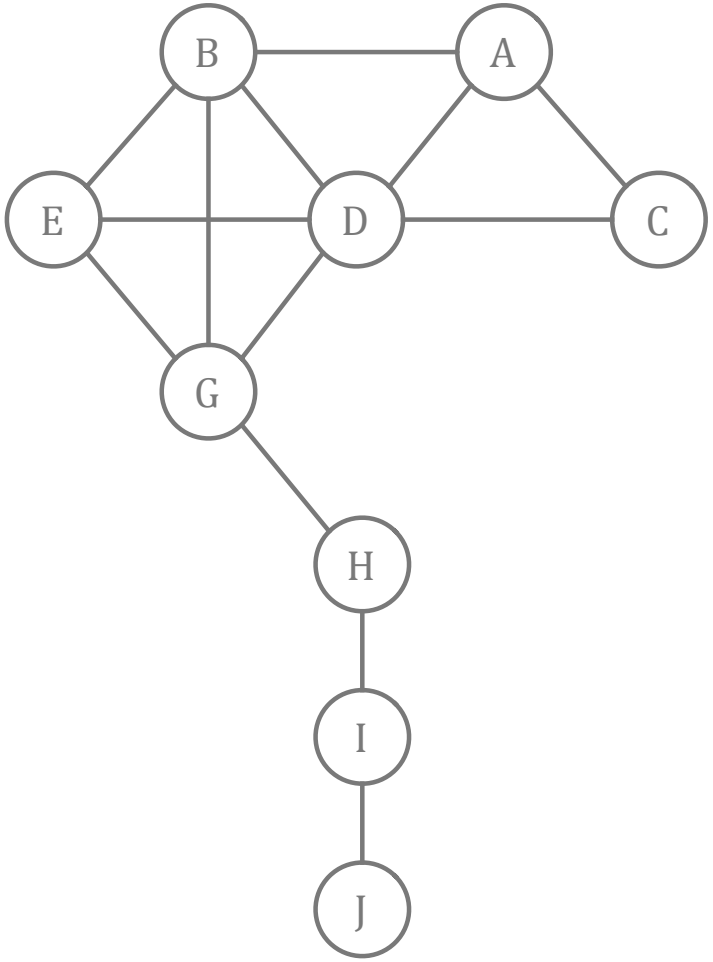

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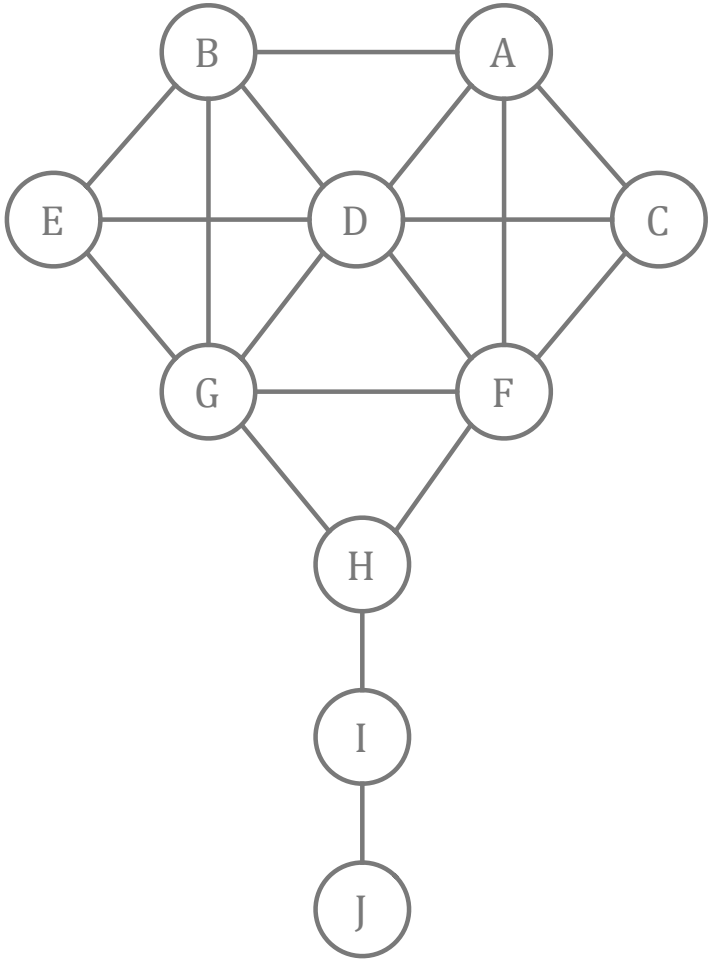
Other Network Measures



- Single points of failure
  - Heather
- Key network paths
  - Fernando, Garth
- Boundary spanners
  - Fernando, Garth, Heather
- Peripheral players
  - Ike, Jane

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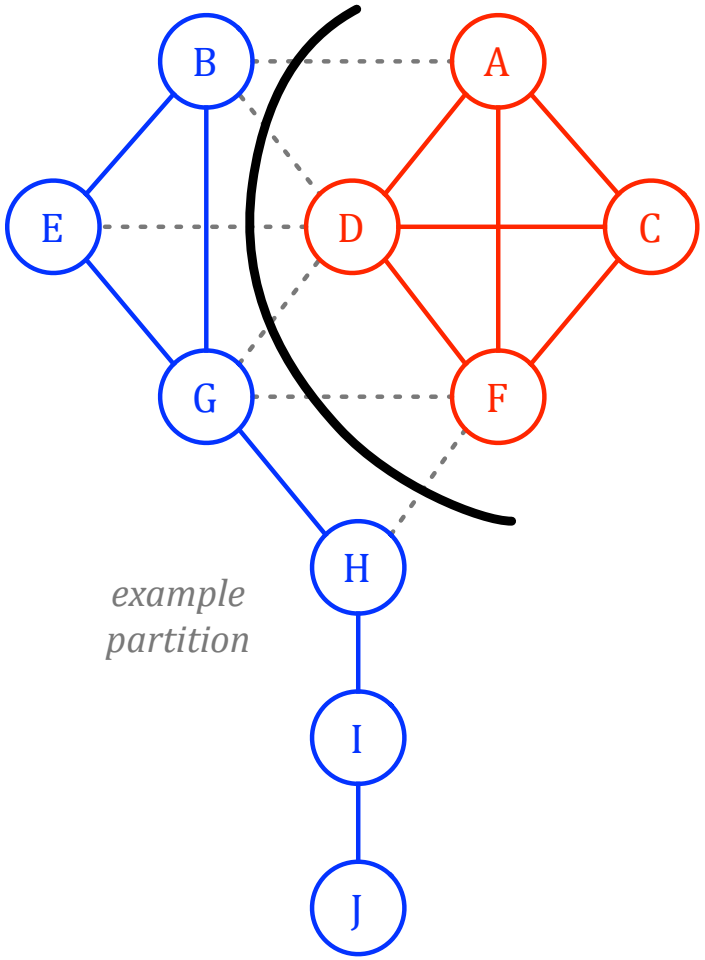
Minimum Cut Set



- Cut
  - Partition of vertices
- Cut Set
  - Set of edges that cross the partition
- Minimum Cut Set
  - Polynomial
- Maximum Cut Set
  - NP Hard

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QUESTIONS?

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